

Drawing on different disciplines: macroeconomic agent-based models

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Abstract Macroeconomic modelling has been under intense scrutiny since the Great Financial Crisis, when serious shortcomings were exposed in the methodology used to understand the economy as a whole. Criticism has been levelled at the assumptions employed in the dominant models, particularly that economic agents are homogenous and optimising and that the economy is equilibrating. In a related paper (Haldane and Turrell, 2018), we argue that an interdisciplinary approach to modelling in macroeconomics is beneficial. Here we focus on what one such approach - agent-based modelling, which has been extensively used across a wide range of disciplines - could do for macroeconomics. Agent-based models are complementary to existing approaches to macroeconomics and are particularly well-suited to answering questions where complexity, heterogeneity, networks, and heuristics play an important role.

Keywords macroeconomics · modelling · agent-based

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The views expressed in this paper are solely those of the authors, and do not necessarily represent those of the Bank of England or its policy committees and should not be reported as such. This paper was initially prepared for a special edition of The Oxford Review of Economic Policy. A modified version is reproduced here with an additional section which examines the barriers to the more widespread adoption of macroeconomic agent-based models.

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1 Introduction

The economic and financial crisis has arguably spawned a crisis in the economics and finance profession. Much the same occurred after the Great Depression of the 1930s when economics was rethought under Keynes' intellectual leadership (Keynes, 1936). The challenge today, to academics and to policymakers, may be every bit as great. A spotlight has been thrown onto the models used prior to the crisis. Critics argue that they were too restrictive and were not supported by empirical evidence. Academic work is beginning to plug these holes and new perspectives are being sought. Rising to the challenge of modelling a broader set of economic circumstances requires macroeconomists to think afresh, perhaps seeking inspiration from other disciplines with different methodological approaches. This paper outlines how a more pluralistic methodology could be beneficial, and discusses how macroeconomic agent-based models could provide one path for rising to these challenges.

The structure of the paper is as follows; §2 discusses why a broader view of what should form the core of macroeconomic models would be beneficial, while §3 considers how agent-based models in economics may differ from similar models in other disciplines. §4 asks what kind of problems macroeconomic agent-based models might help solve, §5 looks at what the barriers might be to their more widespread use and §6 concludes.

2 A broader view of modelling in macroeconomics

Since the 1970s, the benchmark macroeconomic model has been heavily influenced by a single methodology. This began with the 'New Classical Counter Revolution' in which Lucas, Sargent, Kydland and Prescott (Kydland and Prescott, 1982; Lucas and Sargent, 1979) overturned the use of 'structural' or 'policy' modelling of aggregate macroeconomic variables. These structural econometric models, said Lucas and others, had made several mistakes, including saying little about the stagflation of the 1970s and making implausible theoretical assumptions in order to match the data. Lucas' most famous critique of these models was that they were not robust to changes in policy; that they did not allow for agents' behaviours to change as the incentives of those agents changed (Lucas, 1976). Lucas' critique is reasonable in that robust models should seek to explain how changes in policy might affect aggregate outcomes. In practice, no model is fully Lucas critique-proof; it is a matter of degree.

Lucas and others developed a methodology which they believed would not fall foul of the critique. The paradigm which Lucas believed best achieved this grounded macro-economic fluctuations is so-called 'microfoundations'. Simply put, this said that macro-economic behaviour should be built up from the aggregation of the individual actions of self-interested, typically optimising, agents. Being grounded in optimising behaviour, these self-interested behaviours were less susceptible to change when aggregate macroeconomic relationships changed.

In practice, such microfoundations often became closely associated with a particular type of self-interested behaviour, namely optimisation with rational expectations (Lucas, 1972, 1987; Muth, 1961). Coupled with these expectations are assumptions about agent optimality. Typically, this takes the form of an assumption that agents maximise their discounted sum of expected future utilities, subject to a budget constraint.

These microfoundations have a number of desirable properties. They can serve as a useful approximation of real-world behaviour, in at least some situations. They condense the world into a small number of readily-observable factors, which can help in determining what is driving what. And they are sometimes more analytically tractable, and are certainly more analytically elegant, than most of the alternatives.

They do, however, have a number of limitations. For one, they invoke strong assumptions (such as rational expectations and optimisation) which are often not borne out in the data (Estrella and Fuhrer, 2002). One defence of rational expectations is that, even though individuals may not be rational, their irrationalities cancel at the aggregate level. Shaikh (2016) has shown how several different microeconomic behaviours can lead to the same macroeconomic outcomes. But it is well-known from other disciplines that, in general, heterogeneous micro-level behaviour combines to generate complex, non-linear responses and emergent behaviour at the macro-level. There are also simple scenarios where the strong version of rational expectations breaks down, for instance in minority games (Arthur, 2006). In general, there are relatively few cases in which aggregation to the macro-level undertaken in models which use rational expectations are completely sound (Kirman, 1992).

These limitations mean that a broader view on the types of behaviours and expectations which are appropriate for macroeconomic models could help to improve the performance of those models in matching real-world data. This would include models which go beyond the narrow interpretation of the Lucas critique as requiring microfoundations comprising agents that form expectations rationally and who behave optimally, no matter how high the computational cost of doing so.

One plausible alternative set of microfoundations would draw instead on empirically observed behaviours among consumers, firms and governments. The observed behaviours are often termed heuristics or ‘rules of thumb’. These often arise from models which draw on insights from psychology to understand human behaviour (Tversky and Kahneman, 1975). Agent-level analytical descriptions of these behaviours are an active area of research (Gabaix, 2017). What constitutes rationality is itself not clear or well defined (Simon, 1959). Indeed, in a world of Knightian uncertainty (Knight, 2012), imperfect information, altruism, and costly computation, there is an emerging body of evidence suggesting that heuristics are more ‘rational’, at least as measured by performance, than the narrowly-defined rationality in which agents compute their optimal course of action without any limitations to their information gathering or processing ability (Aikman et al, 2014; Assenza et al, 2017; Gode and Sunder, 1993; Haldane and Madouros, 2012; Hommes, 2006). This is sometimes called the “ecological rationality” of heuristics (Gigerenzer and Brighton, 2009).

The models which arguably dominate the rational-expectations-cum-optimisation methodology in macroeconomics are so-called dynamic stochastic general equilibrium (DSGE) models (Smets and Wouters, 2003). In their most stripped down form, these DSGE models have a unique equilibrium with deviations that are small and smooth, no role for stock variables and micro behaviours of agents which can be simply and linearly aggregated into the behaviour of a representative agent with rational expectations. The majority of central banks take the DSGE framework as their starting point, including the Bank of England (Burgess et al, 2013).

There has been much debate over the pros and cons of DSGE models which we do not seek to repeat here (Colander et al, 2009; Fair, 2012; Smith, 2014). Particularly strong criticism has included that their additions resemble the epicycles of the Ptolemaic system of astronomy (Fagiolo and Roventini, 2012, 2017), their representative agents are “stochastic Robinson Crusoes” (Summers, 2002), and that the models as a whole are “post-real” (Romer, 2016). Post-crisis, researchers are adding heterogeneity in agent types, a role for the financial sector, and bounded rationality for some agents. Some models use exogenous shocks with fat-tailed, rather than Gaussian, distributions (Ascari et al, 2015). These are all useful additions. There are earlier rational expectations models with heterogeneity added via probability mass functions; see, for example, (Heathcote, 2005). The originators of the pre-crisis workhorse DSGE model (Smets and Wouters, 2003) recently published a paper containing many of these modifications, including a zero lower bound, non-Gaussian shocks and a financial accelerator. They find that the extensions go (Lindé et al, 2016)

“some way in accounting for features of the Great Recession and its aftermath, but they do not suffice to address some of the major policy challenges associated with the use of nonstandard monetary policy and macroprudential policies”

There are also new research directions, such as the heterogeneous agent New Keynesian DSGE models (Kaplan et al, 2016; Ravn and Sterk, 2016), which feature more than one type of equilibrium.

Despite these important modifications, it seems likely some features of economic systems will remain very difficult to reproduce in a DSGE setting – for example, crisis dynamics. DSGE models have struggled to simultaneously explain the stylised facts observed in real economic systems (Fukac and Pagan, 2006). Table 1 shows a selection of these stylised facts. An important example is the distribution of GDP growth seen historically, which is shown in Figure 1. Around 18% of the data across the UK, US, Germany and Japan fall outside the best-fit normal distribution. DSGE models do not tend to reproduce, except by construction, the excessively large variation in GDP growth seen in the historical data (Ascari et al, 2015).

The Global Financial Crisis provides a good example of these limitations. DSGE models struggled to explain either how it started or how it propagated. Figure 2 shows the range of forecasts for UK GDP growth produced by 27 economic forecasters (including the Bank) in 2007. Pre-crisis forecasts were very tightly bunched in a range of one percentage point. The methodological mono-culture produced, unsurprisingly,

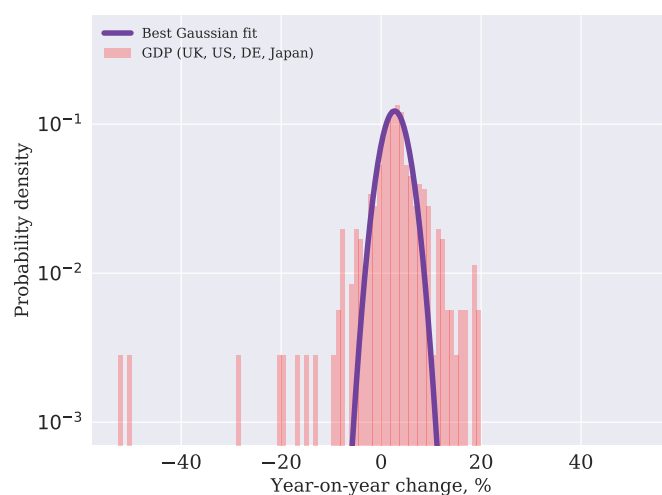


Fig. 1: The distribution of year-on-year growth in GDP, 1871–2015. Source: Hills et al (2016).

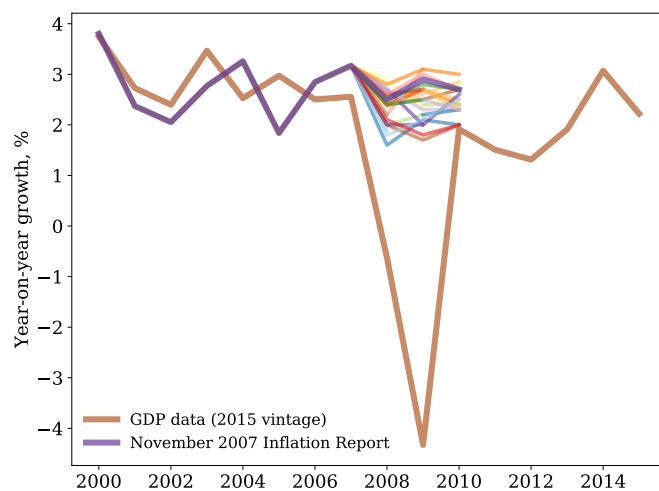


Fig. 2: Range of GDP forecasts in 2007Q4. Source: Haldane (2016).

the same crop. These forecasts foresaw a continuation of the gentle undulations in the economy seen in the decade prior to the crisis – the so-called Great Moderation (Bernanke, 2004). At the time, the damped oscillations of the Great Moderation seemed to match well the smooth motion of DSGE models.

The most important point here is not that this set of models did not forecast the precise timing of the crisis. Almost by definition, costly financial crises cannot be forecast because, if they could, central banks and governments would take actions to prevent

Table 1: Examples of stylised facts.

Stylised fact	Examples
Endogenous self-sustained growth	Burns and Mitchell (1946); Kuznets and Murphy (1966); Stock and Watson (1999); Zarnowitz (1985)
Fat-tailed GDP growth-rate distribution	Castaldi and Dosi (2009); Fagiolo et al (2008)
Recession duration exponentially distributed	Ausloos et al (2004); Wright (2005)
Relative volatility of GDP, consumption and investment	Napoletano et al (2006); Stock and Watson (1999)
Cross-correlations of macro variables	Napoletano et al (2006); Stock and Watson (1999)
Pro-cyclical aggregate research and development investment	Wälde and Woitek (2004)
Cross-correlations of credit-related variables	Leary (2009); Lown and Morgan (2006)
Cross-correlation between firm debt and loan losses	Foos et al (2010); Mendoza and Terrones (2012)
Distribution of duration of banking crises is right skewed	Reinhart and Rogoff (2009)
Distribution of the fiscal costs of banking crises to GDP ratio is fat-tailed	Laeven and Valencia (2013)
Firm (log) size distribution is right-skewed	Dosi et al (2007)
Fat-tailed firm growth-rate distribution	Bottazzi and Secchi (2003, 2006)
Productivity heterogeneity across firms	Bartelsman and Doms (2000); Dosi et al (2007)
Persistent productivity differential across firms	Bartelsman and Doms (2000); Dosi et al (2007)
'Lumpy' investment rates at firm-level	Doms and Dunne (1998)
Counter-cyclicality of firm bankruptcies	Jaimovich and Floetotto (2008)
Firms' bad-debt distribution fits a power law	Di Guilmi et al (2004)
Firm sizes fit a Taylor power law	Gaffeo et al (2012)

them. The real problem was that these models said nothing about the *probability* of a serious crisis arising endogenously at any time, or about the downstream consequences for the economy of a crisis once it had struck. The absence of non-rational expectations, heuristics and non-linear amplification channels was probably key in explaining these problems.

The more general point is that a single model framework is unlikely to best serve the needs of macroeconomists in every state of nature. At its best, the scientific method calls for carefully controlled experiments applied to a rich ecology of models, enabling gradual selection of those models which best fit the known facts. But macroeconomists rarely, if ever, have the luxury of running experiments. And even when they can, experimental model validation is hard because the macroeconomy is a complex system of interacting parts in which cause and effect are difficult to separate.

Facing these constraints, it is likely that a patchwork of models will be more resilient than a single methodology. A group of genuinely distinct models, in competition to match the moments of the real-world, are likely to produce a far richer set of insights

than a single class of models, however aesthetically beautiful. In other words, what may be needed is a “Cambrian Explosion” in macro-economic modelling.

From forecasting, there is evidence that combining two or more models leads to greater predictive power than using one model alone (Silver, 2012; Stock and Watson, 2006; Timmermann, 2006). This has been specifically demonstrated for inflation forecasting by Norges Bank in Bjørnland et al (2012). It is likely to be true qualitatively as well as quantitatively; what one model does not pick up or explain well may be explained by another. A set of models which are distinct, but plausible, will be more informative jointly, perhaps especially so when they disagree. This ‘zoo of models’ approach has also been adopted at the Bank of England (Burgess et al, 2013).

What types of animal are likely to be most useful in the zoo? At a high level, two types: single equilibrium ‘type I’ economic models for dealing with close to equilibrium fluctuations; and more complex, multiple equilibrium ‘type II’ economic models for dealing with far from equilibrium fluctuations. Type I models are stationary and proximately linear. Behaviours are well anchored and close to optimising. Agents’ interactions are predictable and aggregate to something close to a single representative agent (Kirman, 1992; Solow, 2008).

Type II models capture behaviours which are ‘irrational’ or ‘heuristically rational’ and heterogeneous. Uncertainty, as distinct from risk, is acute. Aggregate behaviour in these models is likely to be fat-tailed and often emergent. Type II models are also likely to help educate us by playing out scenarios we did not expect when constructing the model, combining known micro-level features to produce unexpected aggregate outcomes. This is the “if you didn’t grow it, you didn’t explain it” philosophy described in Epstein (1999).

Type II macroeconomic models should be able to explain how economic features, including crises, can arise endogenously. To give a concrete example, a type I analysis of the crisis would invoke an exogenous crisis shock – in which a large number of consumers default on a significant fraction of their loans – to understand how the Great Financial Crisis then evolved (Kumhof et al, 2015). A type II model would show how crises unfold not due to any exogenous shock, but as a natural consequence of the rules and behaviours of the agents within the model of the economic system over time. This provides a mechanism for exploring policies which reduce the frequency or severity of crises.

As Krugman and others have pointed out (Colander et al, 2009; Krugman, 2011), some partial (dis-)equilibrium ways of thinking about the economic system could have helped in understanding the crisis before it unfurled, including the ideas of Bagehot (1873), Leijonhufvud (2000), Kindleberger (2001) and Minsky (2008). Before the New Classical Counter Revolution, structural econometric models were also popular (Blanchard, 2017) - the US Federal Reserve’s FRB/US macro model fits into this category (Brayton and Tinsley, 1996) (see Welfe (2013) for a review of these types of model). However, all of these models tend to operate at the aggregate level and in general equilibrium, rather than aggregating from the agent-level. Because

of that, they are less well-suited to tackling problems with a high degree of agent heterogeneity or with shifting equilibria.

The hegemony of the “representative agent with rational expectations” approach runs deep in macroeconomics. It has similarities to Newtonian physics. But this quasi-mechanistic view, while sometimes a useful approximation, is not a good representation even of modern-day physics. Modern physics research deals with complex systems, emergent behaviours, vast simulations, and outcomes which are probabilistic and stochastic beyond what is implied by the Gaussian distribution. As an example with parallels in economics, aggregate thermodynamic variables like temperature are not sufficient to describe the rich dynamics of systems which are far from equilibrium; granular descriptions of individual particles and their distributions are required (Turrell et al, 2015a). The tools that have been developed for tackling problems that are not analytically tractable (such as complex behaviours and high degrees of heterogeneity) were first developed in physics and thereafter found use in many other natural and social sciences. These same approaches could now offer a complementary approach to modelling in macroeconomics.

Agent-based models have their origins in the 1930s. Physicist Enrico Fermi was trying to solve a particularly difficult problem: the movement of one of the types of particle which make up the atom, the neutron, through background material. This was a tricky calculation as the neutrons had a distribution over several variables including position \mathbf{r} , energy E , and direction $\hat{\Omega}$ according to a distribution $f(\mathbf{r}, E, \hat{\Omega}, t) dr dE d\Omega$.

The full equation tracks the position, direction and energy evolution of N particles over time with terms representing numerous discrete, continuous, source, and sink interactions of the neutrons. Initially, Fermi and other scientists tried to solve the entire problem analytically in single equation form but it proved difficult to solve for all but the simplest cases.

Fermi developed a new method to solve these problems in which he treated the neutrons individually, using a mechanical adding machine to perform the computations for each neutron in turn. Fermi took great delight in astonishing his colleagues with the accuracy of his predictions without, initially, revealing his trick of treating the neutrons individually (Metropolis, 1987).

The more general method of using random numbers to solve problems soon got a name that reflected its probabilistic nature: Monte Carlo. It was further developed by Fermi, Stanislaw Ulam, John von Neumann and others (Metropolis, 1987; Metropolis and Ulam, 1949; Metropolis et al, 1953). It found wide usage because of the way it naturally weights the scenarios that are explored by the probability of them occurring. It is efficient in problems with a high number of dimensions, and effective in reproducing all of the moments of a distribution function. Monte Carlo is a standard technique in finance, where it is used to calculate the expected value of assets.

Monte Carlo simulation remains widely used in a range of disciplines known under different names, including individual-based models in biology and ecology, agent-based models (ABMs) in economics, and multi-agent systems in computer science and logistics as described in Turrell (2016). Recent applications in physics include

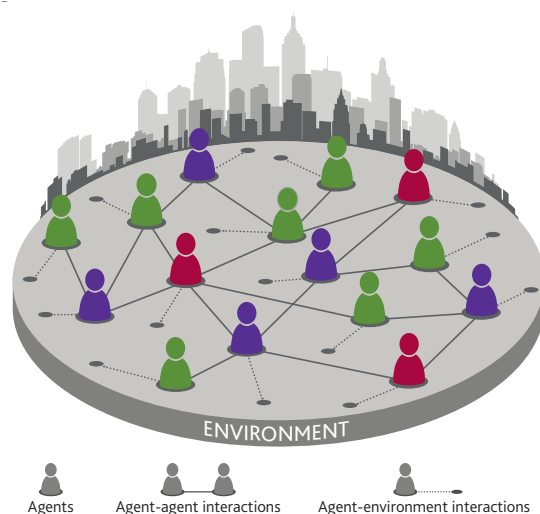


Fig. 3: A schematic of the elements of an agent-based model and their interactions. Source: Turrell (2016).

calculating how beams of particles could destroy cancerous cells (Arber et al, 2015; Bulanov and Khoroshkov, 2002), and how to produce energy from nuclear fusion reactions (Lindl et al, 2004; Spears et al, 2015). They have made a mark in ecology (Carter et al, 2015), where they have been used to model endangered species; in epidemiology (Degli Atti et al, 2008), where they have been used to make detailed predictions of how influenza could spread given demographics and transport links; and for the decentralised behaviour of autonomous vehicles (Ernest et al, 2016). As in physics, their use in epidemiology sees a set of difficult to solve differential equations being replaced with simulations of individual agents. Monte Carlo simulation passes the market test too. It has been used for project finance modelling under uncertainty, forecasting mortgage repayment rates (Geanakoplos et al, 2012), redesigning the rules of the NASDAQ stock exchange (Bonabeau, 2002), and for simulating the transport of people (Heppenstall et al, 2011). A number of firms, such as Sandtable and Concentric, offer bespoke agent-based models for commercial applications.

Agents in these models might include the consumers in an economy, fish within a shoal, and even galaxies within the Universe as in Davis et al (1985). As well as interacting directly with each other, agents might also have a connection to their environment – for instance, banks subject to regulation or whales migrating across the ocean. The behaviours or rules that agents follow depend on the question of interest. Some models have many different types of agent, perhaps firms, workers and governments. These may themselves differ, so that while all workers have a chance to be employed by a firm and receive a wage, the human capital and marginal propensity to consume of each worker could be different and determined according to an empirical distribution. A schematic of an agent-based model in economics is shown in Figure 3.

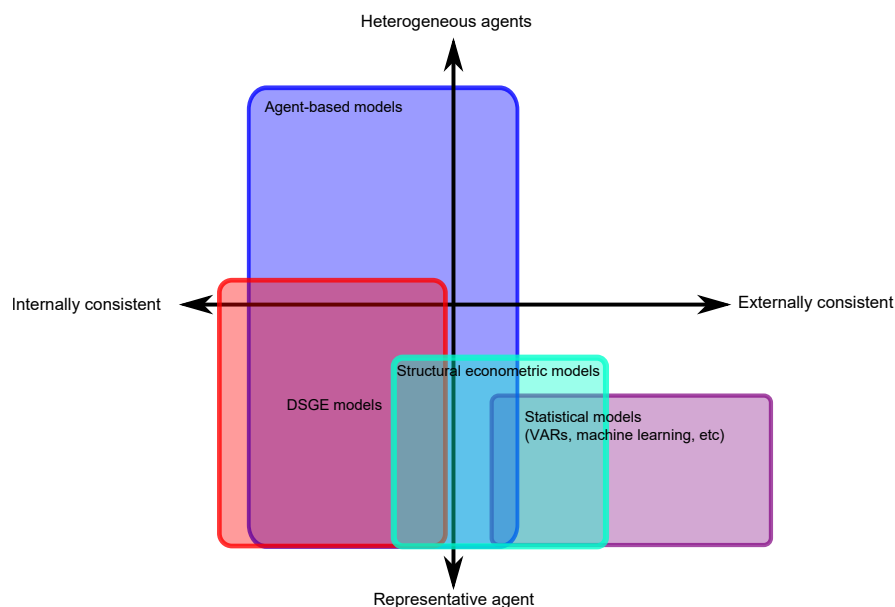


Fig. 4: Macroeconomic ABMs may be thought of as lying within a wider modelling space, here shown as having two axes. Internal consistency is best represented by strongly microfounded behaviour, while external consistency is demonstrated by agreement with the data. On the other axis is the degree of agent heterogeneity which the model can include, with representative agents on one end and heterogeneity along many dimensions at the other. The within variation of each model type is likely to be larger than the variation between them, but the figure illustrates their approximate location within the wider modelling space. Note that VARs represent unstructured vector auto-regressions.

Heterogeneous agents interact both with each other within a network structure, and with the wider environment.

The important feature of ABMs is that they explain the overall evolution of a system by simulating the behaviour of each individual agent within it and then explicitly combining their micro-level behaviours to give a macro-level picture. Each agent is a self-contained unit which follows a given set of behavioural rules. This ‘bottom-up’ approach is very much in the spirit of a “microfoundations” approach, though it differs fundamentally in how it then aggregates to the macro level.

3 How are agent-based models in macroeconomics different?

There are important differences between ABMs in the sciences and in economics. In economics, agent-level behaviours are not known to the same level of accuracy as

the laws of nature which govern the interactions between, for example, particles. In economics, behaviours can change over time in response to the environment. Agent level assumptions in economics thus need to be rigorously tested and varied.

Partly as a consequence of the inherent uncertainty in agent behaviour, ABMs in economics can match the data only probabilistically. They tend to match moments and reproduce stylised facts. Some argue that this is naïve compared to more exactly matching the historical evolution of variables over time. But the latter approach, often using forcing processes, seems to suggest an implausible level of precision. An ABM is a way to generate many possible, plausible realisations of variables in exactly the same manner as different possible price paths are generated in Monte Carlo option pricing. Another way to think of this is in terms of the trade-off between bias and variance. Very broadly, ABMs attain lower bias at the cost of higher variance. Model errors are squared in the bias but only linear in the variance (Friedman et al, 2001). Models with forcing processes aim at low variance at the cost of higher bias.

Where do ABMs fit into the wider modelling landscape? Figure 4 shows this schematically. Models lie on a spectrum. Statistical models rarely say anything about heterogeneous agents. DSGE models say more, and ABMs yet more. But ABMs are not useful for every problem. While in principle it would be possible to use them in forecasting, there are already models potentially better equipped for this, such as dynamic factor models (Stock and Watson, 2011) and machine learning (Chakraborty and Joseph, 2017). ABMs are better placed to produce conditional forecasts, where a particular policy is being explored. This is how epidemiologists use agent-based models too (Degli Atti et al, 2008): rather than attempt to forecast the specific time that a virus outbreak will happen, they identify the risk factors for a virus to break out and subsequently spread. In Figure 4, ABMs nest DSGE models as a special case with little heterogeneity, no stock variables and a particular set of assumptions about agent behaviour. Broadly defined, ABMs will add most value when problems revolve around heterogeneity, complexity, non-linearity, emergence, heuristics, and detailed rules.

The models in Figure 4 require quite different approaches and ABMs have a modelling philosophy which is distinct from their closest neighbours. The archetypal DSGE model comes bundled with a number of assumptions, including rational expectations. Rather than offering a ‘core’ model, ABMs are a flexible toolkit for solving complex problems involving heterogeneous agents. One could use rational expectations, but there is no requirement to do so. This flexibility is one reason why ABMs populate every field from military war games to ecology, and why it is impossible to write down a representative ABM.

As a demonstration of this, Table 2 shows a non-exhaustive list of consumption functions in different macroeconomic ABMs. In DSGE models, a core model has emerged partly because the need for analytical tractability forces the modeller to choose from a restricted set of behaviours. ABMs are a generalisation to more behaviours and more agents. This means ABMs are often bespoke, adapted to the particular question they are answering. For that reason, they have been criticised as ‘black box’. Their bespoke nature means that there is also a bigger cost to understanding them.

Table 2: Examples of consumption used in different macroeconomic agent-based models. For full details see the references.

Consumption model description	Consumption model references	ABM references
Inflation dependent fraction of permanent income; consumption given by $c_t = k_t \hat{y}_t$ where $k_t = k_{t-1} - f(i_t - \mathbb{E}\pi_{t+1} - r_t)$	\hat{y}_t is permanent income from Friedman (1957).	Salle et al (2013)
Aggregate consumption as sum of incomes of all employed and unemployed; $C_t = \sum_{u,e} \sum_i y_{i,t}$	Hand-to-mouth consumers as described in Campbell and Mankiw (1989).	Dosi et al (2010)
Fixed propensities α_1, α_2 out of expected real disposable income and expected real wealth $c_t = \frac{1}{\mathbb{E}p_{t+1}} (\alpha_1 y_t + \alpha_2 W_t)$	Godley and Lavoie (2007)	Caiani et al (2016)
Adaptive (with memory parameter ξ) expectation of income and fixed fraction of wealth (based on buffer-stock); $c_t = \xi y_{t-1} + (1 - \xi)y_t + 0.05W_t$	Carroll (1997, 2009)	Assenza et al (2015)
Concave, monotonically increasing bounded above fraction of real wealth; $c_t = \min \left\{ \left(\frac{y_t}{p_t} \right)^\alpha, \frac{y_t}{p_t} \right\}$ with $0 < \alpha < 1$	Carroll and Kimball (1996); Souleles (1999)	Lengnick (2013)
Wealth growth ΔW , average historical consumption of other households \bar{c}_t and a reference consumption level \bar{c}_t ; $c_t = \bar{c}_t + \alpha \frac{\Delta W_t}{p_{t-1}} + \beta (\bar{c}_{t-1} - c_{t-1})$ where $\alpha > 0, \beta > 0$	Abel (1990); Jawadi and Sousa (2014)	Guerini et al (2016)
Numerically solved utility maximisation subject to inter-temporal budget constraint with leisure $(1 - l)$ and wealth W ; $\max_{c,l} \mathbb{E} \left\{ \sum_{r=t}^{\infty} \beta^{r-t} u(c_r, 1 - l_r) \right\}$ such that $W_{t-1} - p_t c_t > 0, c_t > 0$	Friedman (1957)	Sinitzskaya and Tesfatsion (2015)
Consumption as a fixed fraction of wealth and income; $c_t = \alpha(W_t + y_t)$	Godley and Lavoie (2007)	Gatti and Desiderio (2015); Gualdi et al (2015)
Buffer-stock with mean backward-looking income \bar{y}_t and target wealth-to-income ratio ϕ ; $c_t = \bar{y}_t + \alpha(W_t - \phi \bar{y}_t)$	Carroll (1997)	Chan and Steiglitz (2008); Cincotti et al (2010); Dawid et al (2012, 2014)

Yet there is no reason why an ABM could not have what Olivier Blanchard (2017) has outlined as being required for a core macroeconomic model: nominal rigidities, bounded rationality and limited horizons, incomplete markets and a role for debt. Indeed, these features are found in many macroeconomic ABMs. One of the major differences is the degree to which these models are solved computationally. The process in a typical DSGE model is to specify agents' behaviours and analytically aggregate them, assuming in the process that markets clear. Usually they would then be linearised and, in the last step, solved numerically. In a typical ABM, the process is quite different. They are solved numerically at the agent level, one behaviour at a time.

This has a number of implications for the strengths and weaknesses of ABMs. Being free of the need to specify equations which can fit together and be solved analytically can be liberating for some problems. This does not mean that ABMs cannot be represented mathematically – there is a theorem which says that all ABMs which can be computed numerically have an explicit mathematical representation¹ (Epstein, 2006; Leombruni and Richiardi, 2005). But usually the direct mathematical translation would be too unwieldy to transcribe and would only be interesting in special cases. ABMs are better thought of as algorithms for aggregating the behaviours of individual actors rather than as systems of equations.

If there is a downside to this flexibility it is that analytical certainty must be replaced with numerical convergence. Sinitskaya and Tesfatsion (2015) highlight this by comparing numerical solutions to a lifecycle optimisation problem within an ABM to a known analytical solution. The numerical solution methods do not reach precisely the same outcome as the analytical solution. But for most problems in macroeconomics, the accuracy in how a problem is posed is likely to be a far larger source of error than the lack of precision in the numerical solution of that problem. Indeed, this is another example of the bias-variance trade-off.

The modelling philosophy also differs with respect to interpretation of results. It is useful to think of ABMs as a machine for generating many alternate realisations of the world. Just as in real experiments, it is useful to make many repeats with both control and treatment groups. If the future distribution of one particular variable were truly uncertain, say income y such that $y \sim f$, an ABM could be run not just with millions of possible draws from the same distribution f but also from an entirely different parametric or empirical distribution g . In an ABM, it is possible to do this by changing a single line of code. In more analytical models, it might not be possible at all to do so. Due to the complexity of the interactions within an ABM, it may also be necessary to treat the outcomes as one would real experimental data, with the machinery of hypothesis testing and confidence intervals.

As with all models, it is nonsensical to have more free parameters than imposed or calibrated parameters. ABMs have been accused of being full of free parameters, but this is bad modelling rather than an intrinsic feature of ABMs. One measure of

¹ Every agent-based model is computable by a Turing machine, and every algorithm computable by a Turing machine may be expressed via sets of partial recursive functions.

all models is how parsimoniously they describe the empirical data. Macroeconomic ABMs can match both micro and macro stylised facts for most, if not all, the entries in Table 1. For instance, both Gualdi et al (2015) and Caiani et al (2016) match the frequency and the extent of the cyclicalities in productivity, nominal wages, firms' debt, bank profits, inflation, unemployment, prices and loan losses, while Dosi et al (2015) reproduce, amongst other features, the distributions of output growth and the duration of banking crises. By simulating every individual actor within an economic system, all moments of distribution functions are accessible to ABMs.

If, as described in Wren-Lewis (2016b), DSGE models maintain internal consistency by sacrificing some external consistency, then ABMs are more of a bridge between internal and external consistency: they sacrifice some internal consistency by allowing agents to have behaviours which are not hyper-rational. Nonetheless, the combination of bespoke models, flexibility and a tendency to focus on complex systems can mean that communication is a challenge for agent-based modellers. ABMs are rarely as easy to write down in equation form as, say, the simple three-equation New Keynesian DSGE model. There may not always be an easy fix: complex real systems must sometimes be described with complex simulations, at least initially.

Science has had to deal with the trade-off between bigger, more feature-packed models and more abstract but easier to digest models for a long time. Compromises have emerged. In physics, it is common to use a complex simulation of a system as a way of initially exploring hypotheses or discovering new phenomena. Once a specific effect is identified within the complex simulation, a purely theoretical model (or a much simpler numerical model) is built to explain its salient features. An example is in Turrell et al (2015b), in which a very fast way for lasers to heat matter was first identified in a model with over 10^7 agents and then explained with a handful of parameters in an analytical model using differential equations. Another example of a theoretical model being overturned by simulation is found in (Sherlock et al, 2014). The poor performance of the original theory, accepted for decades, would have been difficult to understand or describe analytically without the use of an ABM.

Gualdi et al (2015) are the exemplars of this approach in a macroeconomic ABM. A rich and complex model is boiled down to a much simpler ABM which retains, and explains, the same phenomenon. The difference between this approach and simply beginning with a smaller, perhaps analytical, model is that the bigger and more realistic model genuinely surprises the researcher with a relationship or phenomenon that they did not expect. It is then the researcher's job to unpick this effect and interrogate it further.

The bottom line is that whether an ABM is good or bad will depend on its specific assumptions, how it is used, and how the results are interpreted. The lack of restrictions on modelling assumptions can be a risk. As it is the flexibility of ABMs that gives rise to this risk, the benefits of this flexibility need to be significant to justify this cost. It is argued in the next section that they are.

4 What could agent-based models do for macroeconomics?

The strength of ABMs for economics lies in their flexibility relative to the other models shown in Figure 4. Ghironi (2018) identifies several topics which are likely to be important for macroeconomic models in the future. They are topics which can be difficult to capture in established models. But, as shown in Table 3, ABMs are already delivering new perspectives on each of them.

In general, ABMs are well-suited to situations where interactions between agents really matter, where heuristics dominate, where the heterogeneity of agents is important, where policies have agent-level implications, where granular data are plentiful, and where analytical methods fail. Non-Gaussian distributions, non-linear equations, time-inconsistent choices, boundedly rational behaviours – it is possible to solve all of these numerically at the agent-level. Given that many real world problems involve these features, ABMs have many potential uses in economics (Tesfatsion, 2002). They can simulate markets which do not necessarily clear, either by having explicit clearing mechanisms, rationing, or emergent frictions.

A great deal of research in networks, particularly as applied to financial systems, has shown that there are emergent properties at the system-level which arise out of interactions at the agent-level (Battiston et al, 2007; Gai et al, 2011). As an example, Bardoscia et al (2017) show that as banks integrate and diversify at the agent-level, they can increase the system-wide risk of instability because they create cyclical dependencies which amplify stress. ABMs can get at this behaviour where agent-level interactions lead to counter-intuitive behaviour at the macro-level. Herding effects (Alfarano et al, 2005) are another example.

One of the most important use cases of ABMs is to explore microfoundations other than rational expectations. Table 2 shows how ABMs run the whole gamut of behavioural assumptions for consumption. There is an emerging body of experimental evidence suggesting that heuristics can outperform ‘rational’ behaviours (Gigerenzer and Brighton, 2009). To have confidence in the conclusions of ABMs, there is a need for a wider body of work on how best to include realistic agent-level behaviours; see Gabaix (2016) for one example of this or Gabaix (2017) for an overview of possible behaviours. And in understanding agent behaviours, advanced machine-learning techniques are likely to be useful in approximating decisions made by real people. Initial steps down this path have already been taken by DeepMind (Leibo et al, 2017).

ABMs are perhaps most easily distinguished from other models by their ability to model heterogeneity (Hommes, 2006). There is mounting evidence of the importance of household, firm and consumer heterogeneity in both status (such as wealth) and behaviour (Alfi et al, 2009; Gabaix, 2011; Guvenen, 2011). A special example is when agents’ stock variables matter. Under the assumptions of the archetypal DSGE model it does not matter that variables such as wealth or debt are not tracked because flows implicitly describe the information about stocks which are relevant for the model. However, there are situations in which richer information on stocks are relevant, such as when heterogeneous agents target a particular level of wealth or debt (Elmendorf

Table 3: Topics identified as being important for the future of macroeconomics (Ghironi, 2018) and the ABMs that study them.

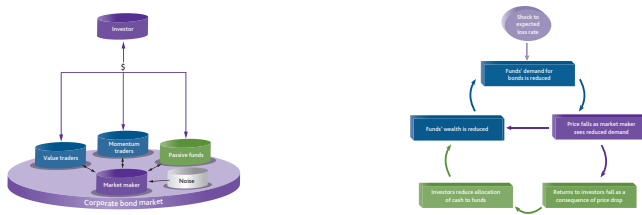
Topic	ABM reference	ABM description
Financial intermediation	Ashraf et al (2017)	Analysis of the role that banks play in firm entry and exit.
Heterogeneity of firms and households	Assenza et al (2016)	Exploration of how exporting and domestic firms become heterogeneous in productivity.
Granularity and networks	Bardoscia et al (2017)	Shows how diversification can lead to increased systemic risk in networks of banks.
Policy interdependence	Popoyan et al (2016)	A macroeconomic model exploring the interdependence between macroprudential regulation and monetary policy.

et al, 1996; Muellbauer and Murata, 2009) or when wealth feeds back into behaviour (Cooper and Dynan, 2014).

Models which include accounting identities at the aggregate level have existed for some time (Wren-Lewis, 2016a). But the recent ABM of Caiani et al (2016) has applied them at the agent level to capture the interaction of agents' decisions through the balance sheet channel. Gatti and Desiderio (2015) show the role of firms' balance sheets in monetary policy. Policies which act heterogeneously also need to be modelled – for instance, the Bank of England Financial Policy Committee's policy in 2015 to ensure “that mortgage lenders do not extend more than 15% of their total number of new residential mortgages at loan to income ratios at or greater than 4.5”. For monetary policy, one of the important practical channels for influencing consumption relies for its effectiveness on agent heterogeneity: those who gain from policy easing have higher marginal propensities to consume than those who lose (Auclert, 2015). When chair of the US Federal Reserve, Janet Yellen commented that (Yellen et al, 2016)

“Economists' understanding of how changes in fiscal and monetary policy affect the economy might also benefit from the recognition that households and firms are heterogeneous. For example, in simple textbook models of the monetary transmission mechanism, central banks operate largely through the effect of real interest rates on consumption and investment. Once heterogeneity is taken into account, other important channels emerge. For example, spending by many households and firms appears to be quite sensitive to changes in labor income, business sales, or the value of collateral that in turn affects their access to credit—conditions that monetary policy affects only indirectly. Studying monetary models with heterogeneous agents more closely could help us shed new light on these aspects of the monetary transmission mechanism.”

An ABM could be used to look at how heterogeneity along several inter-related dimensions affects policy transmission – for instance, marginal propensity to consume as a function of net assets and demographics. Inequality is one type of heterogeneity which is already being explored; see Gibson (2007) and Caiani et al (2016) for examples where initially egalitarian distributions, such as income, become unequal



(a) A schematic of the ABM of trading in the corporate bond market. Source: Braun-Munzinger et al (2016).
 (b) A schematic of how initial price and yield changes are amplified and propagated in the model. Source: Braun-Munzinger et al (2016).

Fig. 5

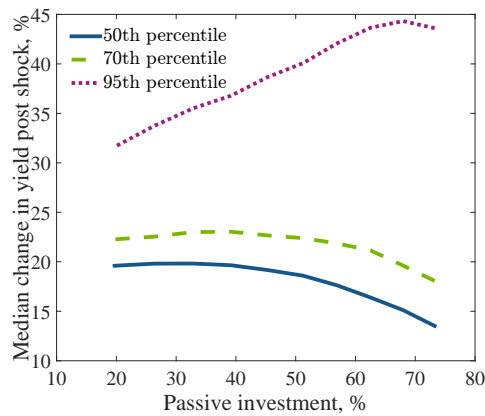


Fig. 6: The distribution of outcomes for the median yield of a corporate bond index over the 100 trading days after a sudden increase in the loss rate on the index. Percentiles refer to repeated model runs. Source: Braun-Munzinger et al (2016).



(a) A schematic of the agents and interactions in the housing market model. Source: Baptista et al (2016).
 (b) A benchmark run of the housing market model showing house price index cycles. Source: Baptista et al (2016).

Fig. 7

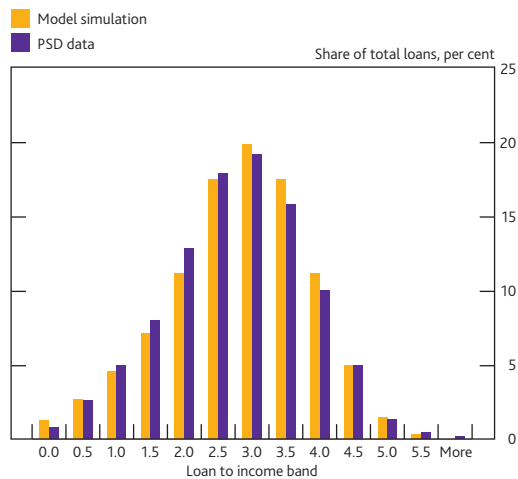


Fig. 8: Once calibrated, the housing market ABM reproduces the loan to income probability mass function for the United Kingdom. Source: Baptista et al (2016).

endogenously. ABMs are not the only way to include heterogeneity, but they do offer an easier route to a higher degree of heterogeneity. Just as Fermi and his colleagues found with neutrons, there is a point where the trade-off between increasingly elaborate mathematical models and numerically solved models favours the latter. New types of DSGE model are closer to that cross-over point, but the other side of the trade-off remains worthy of attention.

The ability to model systems without analytical constraints makes some problems easier to study. Lord Stern, author of a significant review of the economics of climate change (Stern, 2007), has called for agent-based modelling as a way to more realistically incorporate the macroeconomic trade-offs of climate change (Stern, 2016). See Lamperti et al (2017a) for an example of a macroeconomic ABM which takes up this challenge. It facilitates the analysis of systems which are out of equilibrium – systems where markets do not necessarily clear, which are in dynamic dis-equilibrium or which transition between different equilibria. One of the unique perspectives from this type of model is showing how a dis-equilibrium can emerge as the result of the self-interested (but not necessarily hyper-rational) choices of individual agents.

A dramatic example is the non-linear, dynamic macroeconomic ABM of Gualdi et al (2015) in which output and employment discontinuously collapse to new values depending on the propensity of firms to hire and fire new staff, in addition to firms' levels of indebtedness. These discontinuous shifts from one equilibrium to another are well known from physics as 'phase transitions'. They have also been identified in contagion in financial networks (Gai and Kapadia, 2010; Watts, 2002) and in the way that opinions and narratives can shift within a population (Shiller, 2017; Sornette, 2014). De Grauwe (2010) develops an ABM that includes this opinion shift effect in an otherwise New Keynesian setup.

There are undoubtedly problems too. Calibration, for instance, is currently a relatively weak area for macroeconomic ABMs. Although the best have been successful in reproducing an impressive range of stylised facts, calibration techniques and standards tend to vary substantially across models. The generalised method of moments is a commonly used approach (Franke and Westerhoff, 2012), but techniques based on vector auto-regressions and machine learning are also being developed (Guerini and Moneta, 2017; Lamperti et al, 2017b).

The promise of macroeconomic ABMs comes partly from what this approach has already delivered in partial (as opposed to general) economic and financial systems. ABMs have provided plausible explanations for phenomena in financial markets including fat tails, clustered volatility and bubbles (Alfi et al, 2009; Cutler et al, 1989; Hong and Stein, 1999; Lux and Marchesi, 1999). Lux and Marchesi (1999) showed that in order to reproduce the fat tails seen in the distribution of absolute returns in markets, a number of market participants who trade based not on fundamentals, but based on optimism and pessimism (known as noise traders) are required. In their model, agents are able to switch groups based on the performance of strategies. The market fundamentals follow a Gaussian distribution so that the fat-tailed distribution of returns is solely due to the interactions between the different types of traders. In the model, if agents see an opportunity to profit by becoming noise traders, they switch strategies. In the short term, they can ride a wave of optimism or pessimism, and so enjoy larger absolute returns than the variation in fundamentals would imply. But the deviation from underlying value has limits because of the remaining fundamentalist agents and, eventually, prices must partially revert. This simple agent-based model provides a compelling explanation for one of the puzzles of financial markets. At the Bank of England, ABMs have been used to aid understanding of both the corporate bond market (Braun-Munzinger et al, 2016) and the UK housing market (Baptista et al, 2016). The former model, shown schematically in Figure 5a, was used to study how investors redeeming the corporate bonds held for them by open-ended mutual funds can cause feedback loops in which bond prices fall. The non-linear loop induces price overshoots in the bond index (shown schematically in Figure 5b) and the model looks at possible ways to reduce the extent of the overshooting. The model is calibrated to granular empirical data, and produces a reasonable match to the distribution of daily log-price returns. In one scenario, the fraction of funds using passive trading strategies is increased. The increased presence of passive funds is price and yield stabilising in the median case but it introduces a tail-risk of very large price falls (yield rises), as shown in Figure 6. The reason is that when there are few actively trading funds, the market-maker is more likely to observe a sudden glut of positive or negative net demand and so create major price movements in response. This was not a result which was expected *a priori*.

A UK housing market model developed at the Bank of England explores the links between macroeconomic stability and house price cycles (Erlingsson et al, 2014; Geanakoplos et al, 2012). Capturing these cyclical dynamics is not straightforward. One potential reason is that the housing market comprises many types of agent: renters, first-time buyers, owner-occupiers, sellers, and buy-to-let landlords. These agents are all represented in the ABM, and are heterogeneous by age, bank balances,

income, gearing and location. Additionally there is a banking sector (a mortgage lender) and a central bank; all are shown schematically in Figure 7a. The combination of the actions of these agents gives rise to the cyclical dynamics seen in Figure 7b. These arise endogenously in the model.

The inclusion of an explicit banking sector is important as banks provide mortgage credit to households and set the terms and conditions available to borrowers. The lending decisions of the banking sector are subject to regulation by the central bank, which sets loan to income, loan to value, and interest cover ratio policies. The model reproduces the probability mass function of the share of loans by loan-to-income band given by the Product Sales Database of UK mortgages, as shown in Figure 8. The model has been used to look at several scenarios, including policy scenarios. For example, a policy which applies at the agent-level – such as no more than 15% of new mortgages being at loan-to-income ratios at or greater than a given multiple – serves to dampen boom and bust cycles in house prices at the aggregate level.

5 Adopting macroeconomic agent-based models

If macroeconomic agent-based models hold promise for particular sets of question, why haven't they been more widely adopted so far? There are a number of potential barriers. Perhaps the most obvious is that macroeconomic ABMs have not had the same investment in them as some other modelling frameworks. They have simply entered the race later than other approaches and are now playing catch up.

Another factor which may have held back their adoption is that macroeconomic ABMs have tended to be positioned as offering a complete overhaul of the benchmark New Keynesian DSGE model. While we have argued that a wider set of perspectives on modelling the macroeconomy is welcome, acceptance of macroeconomic ABMs might be accelerated if their value relative to existing models could be made clearer. Thus it would be useful to have some, but not all, macroeconomic ABMs able to come closer to matching the benchmark DSGE model in some ways. Hybrid models which could effectively form a bridge between the two approaches, with ABM features which can be turned on and off within a numerically solved framework, would be a useful addition to the literature.

There are already some positive possible threads to follow here. There are already ABMs which have properties that lead them to behave more like conventional models in some settings (Salle et al, 2013; Sinitskaya and Tesfatsion, 2015). There are also recursive models which have ABM-like features; for instance Aiyagari (1994), in which households are heterogeneous in that they face idiosyncratic income shocks. Models of 'epidemiological expectations', in the spirit of Carroll (2001), could replace conventional rational expectations assumptions with a more realistic model of expectations-formation and propagation across heterogeneous set of agents. Gobbi and Grazzini (2017) present work which is truly a bridge between the DSGE and ABM approaches; it explores the consequences of changing agents' information sets

in a standard New Keynesian setup following Galí (2015). These are, in effect, hybrid or cross-over models.

Recent developments in DSGE models are further narrowing the gap with ABM approaches. The Heterogeneous Agent New Keynesian (HANK) model of Kaplan et al (2016) has exogenous heterogeneity in the labour productivity of households. Alongside both illiquid and liquid asset classes, this endogenously allows for wealthy, hand-to-mouth households. In García-Schmidt and Woodford (2015), it is shown that a slight relaxation of rational expectations in which they “model the cognitive process by which one imagines people to arrive at particular expectations” causes the effects of forward guidance to be less inflationary than the usual perfect foresight equilibrium would imply. Ilut and Schneider (2014) develop a New Keynesian model which tries to capture the effects of Knightian uncertainty.

The unique features of ABMs, like endogenous heterogeneity along many dimensions, emergence, and dis-equilibrium, remain some way from the frontier of DSGE models. This can form a barrier in itself. Relatedly, understanding these features requires strategies and approaches that may be unfamiliar to many macroeconomic researchers, as we note in §3. Two of the most useful may be especially unfamiliar. The first is to begin with a feature-rich model which has all the possible ingredients to cover the question of interest and to gradually remove the non-salient features. This approach can be justified when, *a priori*, it is not always clear which model features will be salient and there may be channels which the researcher did not foresee, but which are valuable for understanding. It is, however, at odds with the parsimonious, micro-based approaches typical in much of modern macro-economics.

A second strategy is to summarise the most salient features of a rich and complex model with a parsimonious analytical theory. In this way, simulation can inform theory, rather than the other way around. Both of these strategies help to reduce the complexity of the results from agent-based models, while retaining the benefits of including a higher level of complexity initially. But they require a radically different philosophy to building models from the conventional top-down, theory-ahead-of-simulation, approach.

To see the utility of the alternative, bottom-up approach, consider building a small model initially and relying on intuition alone to determine which model features to include. This will sometimes be appropriate. But sometimes simulations will reveal features not conceived of by researcher intuition. For example, the breakthrough AlphaGo Zero algorithm comprises an agent that learns by playing against itself based on only an objective function and a given set of rules (Silver et al, 2017). A variant of these algorithms is likely one day to be used as the brains of agents in ABMs. They can be calibrated to be more realistically representative of human behaviour than the infinitely-lived agent with perfect foresight, which is the stock-in-trade of DSGE approaches. In the shorter term, ABMs provide a ready-simulated laboratory for the latest analytical theories of microeconomic behaviour which embed the sorts of biases and heuristics we know exist in the real world (Gabaix, 2017).

While greatly improved computer hardware and software are creating opportunities for new directions in ABMs, they also present one of the biggest challenges. Familiarity with programming in general purpose languages is not widespread among economists, and ABMs do require a certain level of programming ability. But there are a number of projects, such as ‘HARK’ and ‘Mesa’, which aim to provide flexible toolkits that make it easy to create ABMs by giving off-the-shelf access to agent types and behaviours. Schools and universities are also including more programming on their syllabuses. Wider collaboration on computer code, in the form of code comparison workshops (Rubiano et al, 2007) and open source toolkits, have been fruitful in the natural sciences. Economics would likely benefit from them too.

These barriers – investment, the lack of a bridge to existing models, new modelling strategies and infrastructure – are ones that it will take time to lower. Yet optimism is warranted. This special issue is itself evidence of that. But what might act as the catalyst for a more decisive breakthrough by ABMs?

As with all modelling, predicting an effect that is later discovered to occur empirically is compelling evidence of model utility. ABMs in finance have found more widespread acceptance because they have successfully explained otherwise puzzling phenomena. An ABM that gave a unique and unambiguous explanation of a puzzling phenomenon in macroeconomics would be similarly hard to ignore. Or perhaps an ABM could provide a big marginal insight in a comparative study with another model, giving the same results in one limit but telling us something new in another. Even a little irrationality or heterogeneity, can go a long way in changing macroeconomic outcomes.

6 Conclusion

The type of microfoundation embedded within mainstream macroeconomic models since the 1970s is far from the only, or in some cases the most plausible, choice. The evidence suggests that a more diverse approach to macroeconomic modelling may be beneficial when making sense of the economy and when setting policy to shape the economy.

Drawing on the last seven decades of simulation, macroeconomic agent-based models are one approach with promise. In these models, aggregate behaviour emerges from the behaviours of individual agents and, in this sense, they are very much in the spirit of the “microfoundations revolution”, even if the modelling philosophy is different to that of the dominant macroeconomic models. Having already proven their worth in partial (dis)-equilibrium settings both in finance and the natural sciences, agent-based models are well placed to allow researchers to study heterogeneity, networks and crisis dynamics within a macroeconomic context. There is work to be done before macroeconomic ABMs enjoy the same standing in mainstream economics as their cousins. But the literature thus far suggests that they could be an important complement to the current crop of macroeconomic models, especially when making sense

of the types of extreme macro-economic movements the world has witnessed for the past decade.

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Compliance with Ethical Standards:

The authors declare that they have no conflict of interest.

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